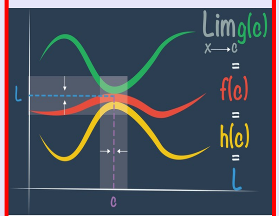
Calculus I

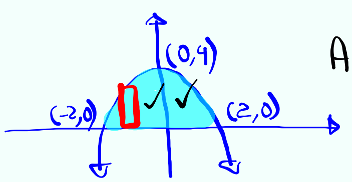
Lecture 48



Feb 19-8:47 AM

Class Quiz 18

Find the area below $\boxed{f(x)=4-x^2}$ and above the $\boxed{x\text{-axis}}$ Drawing Required Box Your Final Ans



$$A=\int_{-2}^{2}(4-x^2-0)\,dx$$

$$=2\int_{0}^{2}(4-x^2)\,dx$$

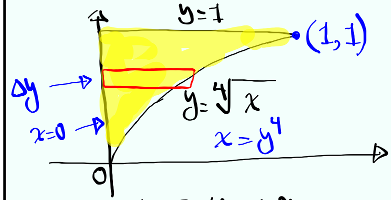
$$=2\left[4x-\frac{x^3}{3}\right]\Big|_0^2$$

$$=2\left[4\cdot 2-\frac{2^3}{3}-0\right]$$

$$=2\left(8-\frac{8}{3}\right)=2\cdot\frac{16}{3}=\boxed{\frac{32}{3}}$$

Nov 26-7:19 AM

Consider the shaded area below (Not Scaled)



Right $x = y^4$

Left $x = 0$

$$A = \int_0^1 [\overset{\text{Right} - \text{left}}{y^4 - 0}]\,dy = \frac{y^5}{5}\Big|_0^1 = \frac{1^5}{5} - 0 = \boxed{\frac{1}{5}}$$

Do this on your own

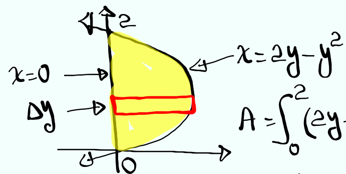
Find the shaded area below

$x = 0$

$2y - y^2 = 0$

$y(2-y) = 0$

$y = 0\ ,\ y = 2$



$$A = \int_0^2 (2y - y^2 - 0)\,dy$$

$$= \left(y^2 - \frac{y^3}{3}\right)\Big|_0^2 = 4 - \frac{8}{3} = \frac{12}{3} - \frac{8}{3}$$

$$= \boxed{\frac{4}{3}}$$

Nov 25-7:49 AM

Evaluate $\displaystyle\int_{-1}^{2} \overbrace{x^2\sqrt{x+2}}^{f(x)}\,dx$

Defined $(-\infty,\infty)$

$x+2 \geq 0$

$x \geq -2$

$[-2, \infty)$

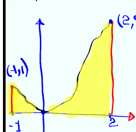
$f(-1) = (-1)^2\sqrt{-1+2} = 1\sqrt{1} = 1$

$f(2) = 2^2\sqrt{2+2} = 4\sqrt{4} = 8$

$x^2 \geq 0$

$\sqrt{x+2} \geq 0 \Rightarrow f(x) \geq 0$

above x-axis



Find the area below $f(x) = x^2\sqrt{x+2}$, above x-axis from $x=-1$ to $x=2$

$$\int_{-1}^{2} x^2\sqrt{x+2}\,dx$$

$u = x+2 \rightarrow u-2 = x$

$du = dx$

$x = -1 \rightarrow u = 1$

$x = 2 \rightarrow u = 4$

$$\int_1^4 (u-2)^2\sqrt{u}\,du$$

$$= \int_1^4 [u^2 - 4u + 4]\sqrt{u}\,du$$

$$= \int_1^4 \left(u^2 \cdot u^{1/2} - 4u \cdot u^{1/2} + 4u^{1/2}\right)du$$

$$= \int_1^4 \left(u^{5/2} - 4u^{3/2} + 4u^{1/2}\right)du$$

Make sure to finish this

$= \frac{562}{105}$

$$\left[\frac{u^{7/2}}{7/2} - \frac{4u^{5/2}}{5/2} + \frac{4u^{3/2}}{3/2}\right]\Bigg|_1^4$$

$$= \left[\frac{2}{7}u^3\sqrt{u} - \frac{8}{5}u^2\sqrt{u} + \frac{8}{3}u\sqrt{u}\right]\Bigg|_1^4$$

$$= \left[\frac{2}{7}\cdot 4^3\cdot\sqrt{4} - \frac{8}{5}\cdot 4^2\sqrt{4} + \frac{8}{3}\cdot 4\sqrt{4}\right] - \left[\frac{2}{7} - \frac{8}{5} + \frac{8}{3}\right]$$

$$= \frac{256}{7} - \frac{2}{7} - \frac{256}{5} + \frac{8}{5} + \frac{64}{3} - \frac{8}{3}$$

$$= \frac{254}{7} - \frac{248}{5} + \frac{56}{3} = \boxed{\frac{562}{105}}$$

Nov 25-8:08 AM

Evaluate $\int_{-1}^{2} x^2 \boxed{\sqrt{x+2}}\,\boxed{dx}$

$u = \sqrt{x+2}$

$u^2 = x + 2 \rightarrow u^2 - 2 = x$

$2u\,du = dx$

$x = -1 \rightarrow u = 1$

$x = 2 \rightarrow u = 2$

$$\int_1^2 (u^2-2)^2 u \cdot 2u\,du$$

$$= 2\int_1^2 (u^4 - 4u^2 + 4)\cdot u^2\,du$$

$$= 2\int_1^2 [u^6 - 4u^4 + 4u^2]\,du$$

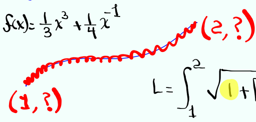
$$= \frac{562}{105}$$

Finish this and compare to last answer.

Nov 25-8:08 AM

Find the arc length of the curve given by $f(x) = \frac{x^3}{3} + \frac{1}{4x^1}$ or $1 \le x \le 2$.

$f(x) = \frac{1}{3}x^3 + \frac{1}{4}x^{-1}$



$$L = \int_1^2 \sqrt{1 + [f'(x)]^2}\,dx$$

$f'(x) = \frac{1}{3}\cdot 3x^2 - \frac{1}{4}x^{-2} = x^2 - \frac{1}{4}x^{-2}$

$1 + [f'(x)]^2 = 1 + \left(x^2 - \frac{1}{4}x^{-2}\right)^2$

$= 1 + x^4 - 2\cdot\cancel{x^2}\cdot\frac{1}{4}\cancel{x^{-2}} + \frac{1}{16}x^{-4}$

$= 1 + x^4 - \frac{1}{2} + \frac{1}{16}x^{-4}$

$= x^4 + \frac{1}{2} + \frac{1}{16}x^{-4} = \left(x^2 + \frac{1}{4}x^{-2}\right)^2$

$\sqrt{1 + [f'(x)]^2} = \sqrt{\left(x^2 + \frac{1}{4}x^{-2}\right)^2} = x^2 + \frac{1}{4}x^{-2}$

$\int_1^2 \left(x^2 + \frac{1}{4}x^{-2}\right)dx = \frac{59}{24}$

Make Sure to finish it.

$= \left(\frac{x^3}{3} + \frac{1}{4}\cdot\frac{x^{-1}}{-1}\right)\Big|_1^2 = \left[\frac{x^3}{3} - \frac{1}{4x}\right]\Big|_1^2$

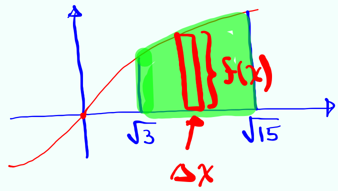
$= \left(\frac{8}{3} - \frac{1}{8}\right) - \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{7}{3} - \frac{1}{8} + \frac{2}{8} = \frac{7}{3} + \frac{1}{8}$

$= \frac{56+3}{24}$

$= \boxed{\frac{59}{24}}$

Nov 25-8:27 AM

Find the area below $f(x) = \dfrac{x}{\sqrt{x^2+1}}$, above the x-axis from $x = \sqrt{3}$ to $x = \sqrt{15}$.



$$f(-x) = \frac{-x}{\sqrt{(-x)^2+1}} = \frac{-x}{\sqrt{x^2+1}} = -f(x)$$

$f(x)$ is odd.
Symmetric → $(0,0)$

$$A = \int_{\sqrt{3}}^{\sqrt{15}} \left(\frac{x}{\sqrt{x^2+1}} - 0 \right) dx$$

$x = \sqrt{3} \to u = 4$

$x = \sqrt{15} \to u = 16$

$u = x^2 + 1$

$du = 2x\,dx$

$\dfrac{du}{2} = x\,dx$

$$= \int_4^{16} \frac{1}{\sqrt{u}} \frac{du}{2}$$

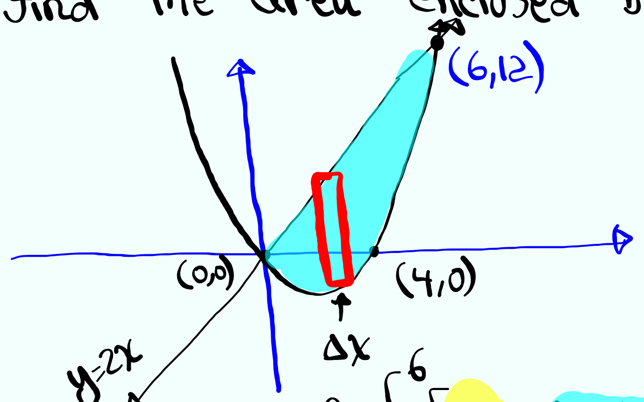
$$= \frac{1}{2}\int_4^{16} u^{-1/2}\,du = \frac{1}{2} \cdot \frac{u^{1/2}}{1/2}\Bigg|_4^{16} = \sqrt{u}\Big|_4^{16} = \sqrt{16} - \sqrt{4}$$

$$= 4 - 2$$

$$= \boxed{2}$$

Nov 26-7:57 AM

Find the area enclosed by $y = 2x$ & $y = x^2 - 4x$.



$x^2 - 4x = 2x$

$x^2 - 4x - 2x = 0$

$x^2 - 6x = 0$

$x = 0 \quad , x = 6$

$$A = \int_0^6 [\text{Top} - \text{Bottom}]\,dx$$

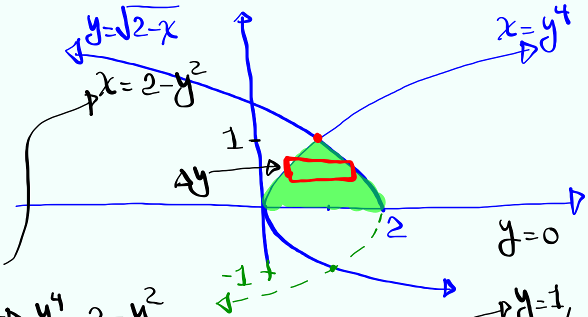
$$= \int_0^6 [2x - (x^2 - 4x)]\,dx = \int_0^6 (6x - x^2)\,dx$$

$$= \boxed{}$$

Nov 26-8:08 AM

Find the area enclosed by $x=y^4$, $y=\sqrt{2-x}$ and $y=0$.

x-axis



$y=\sqrt{2-x}$

$y^2=2-x$

$x=2-y^2$

$x=y^4 \Rightarrow y^4=2-y^2$

$y^4+y^2-2=0$

$(y^2+2)(y^2-1)=0 \rightarrow y=1,\ y=-1$

has no real Solns.

$$A=\int_0^1 [\text{Right} - \text{left}]\,dy = \int_0^1 [2-y^2-y^4]\,dy$$

$$=\left(2y-\frac{y^3}{3}-\frac{y^5}{5}\right)\Big|_0^1 = 2-\frac{1}{3}-\frac{1}{5}=\boxed{\frac{22}{5}}$$

Nov 26-8:15 AM

Find the arc length of $f(x)=\frac{1}{4}x^2-\frac{1}{2}\ln x$ over $[1,2]$.

$f'(x)=\frac{1}{4}\cdot 2x-\frac{1}{2}\cdot\frac{1}{x}$

$f'(x)=\frac{1}{2}x-\frac{1}{2}\cdot\frac{1}{x}$

$$L=\int_1^2 \sqrt{1+[f'(x)]^2}\,dx$$

$$1+\Big[\underbrace{\frac{x}{2}}_{A}-\underbrace{\frac{1}{2x}}_{B}\Big]^2 = 1+\underbrace{\frac{x^2}{4}}_{A^2}\underbrace{-\cancel{2}\cdot\frac{\cancel{x}}{\cancel{2}}\cdot\frac{1}{2\cancel{x}}}_{-2AB}\underbrace{+\frac{1}{4x^2}}_{+B^2}$$

$$=1+\frac{x^2}{4}-\frac{1}{2}+\frac{1}{4x^2}$$

$$=\frac{x^2}{4}+\frac{1}{2}+\frac{1}{4x^2}=\left(\frac{x}{2}+\frac{1}{2x}\right)^2$$

$$L=\int_1^2\sqrt{\left(\frac{x}{2}+\frac{1}{2x}\right)^2}\,dx=\int_1^2\left(\frac{x}{2}+\frac{1}{2x}\right)dx$$

$$=\frac{1}{2}\int_1^2\left(x+\frac{1}{x}\right)dx=\frac{1}{2}\left[\frac{x^2}{2}+\ln x\right]\Big|_1^2$$

$$=\frac{1}{2}\left[\left(\frac{4}{2}+\ln 2\right)-\left(\frac{1}{2}+\cancelto{0}{\ln 1}\right)\right]$$

$$=\frac{1}{2}\left[2+\ln 2-\frac{1}{2}\right]=\frac{1}{2}\left[\frac{3}{2}+\ln 2\right]$$

Nov 26-8:24 AM